# Factor Models for Credit Correlation

Stewart Inglis and Alex Lipton
Merrill Lynch
stewart_inglis@ml.com, alex_lipton@ml.com

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**Abstract**

In this paper we briefly describe dynamic and static factor models for credit correlation, and show how the static model can be calibrated to the market and used for the pricing of standard and bespoke tranches including tranchelets.

## 1 Introduction

Without fear of exaggeration one can say that the structured credit business in general (and credit derivatives in particular) has proven to be one of the most successful financial innovations in recent memory. According to a recent BBA survey, the size of the market at the end of 2006 was about \$30 trillion. Key market participants are banks (trading) (35%), hedge funds (32%), banks (loans) (9%), mono-line insurers (8%), and others (16%). The main types of credit derivatives are credit default swaps (CDSs) (33%), full index trades (30%), bespoke synthetic collateralized default obligations (CDOs) (16.2%), index tranches (7.6%), credit linked notes (CLNs) (3.2%), and others (10%). In essence, CDSs represent insurance contracts on individual obligors, synthetic CDOs represent various tranches of baskets backed by CDSs, while cash CDOs are backed by corporate bonds, mortgages, and other assets. Some of the synthetic tranches are standardized (such as the CDX, iTRAXX and LCDX tranches); others are bespoke OTC instruments created on demand.

Non-linear multi-name products, such as tranches of standardized and bespoke baskets, clearly depend on credit correlation; however, its proper modeling has proven to be one of the more challenging problems of mathematical finance. Without being unduly pessimistic, one can say that an adequate framework for describing credit correlation is still missing. The reasons for the modeling difficulties are manifold, but high dimensionality of the problem is clearly one of them.

Originally, the Gaussian Copula approach was used to price CDO tranches (see, e.g., Li (2000)). Unfortunately, in its basic form it is incapable of reproducing market prices. Several researches have tried to generalize it with some degree of success, see. e.g., Andersen & Sidenius (2004), and Hull & White (2006). These attempts are nicely summarized in a recent review paper by Andersen (2005).

Currently, the so-called base correlation framework is used by most (but not all) investment banks for the pricing and risk management of credit baskets and their tranches. While sufficiently flexible to reproduce the quoted break-even coupons (BECs) for the standard CDX and iTRAXX tranches, this framework is conceptually unsound and its usage can result in unexpected (and often unpleasant) P&L surprises. Pricing of bespoke tranches in this framework is even more problematic. However, the true weakness of the base correlation approach becomes apparent when it is applied to the pricing of dynamic products, such as forward starting tranches and the like.

There exists an obvious temptation to reduce the dimensionality of the correlation problem by assuming that the evolution of different credits in the basket depends on just of a handful of common factors plus (possibly) some idiosyncratic factors which can be integrated out. This can be done in two complementary ways: (A) by modeling the dynamics of hazard rates, cf. Duffie & Garleanu (2001), Mortensen (2006), Chapovsky *et al.* (2007); (B) by modeling the evolution of the so-called latent factors, see, e.g., Baxter (2007), among others.

In the present paper we propose a single latent factor model which provides a satisfactory description of the evolution of individual names as well as their collective behavior. We also build a static version of the model which we apply to solving problems static in nature, such as the pricing of standardized and bespoke tranches, etc.

The paper is organized as follows. In Section 2 we briefly describe a structural model for a single name CDS, which we use as a prototype for developing a multi-name model. In Section 3 we present a dynamic model for solving the pricing problem for a tranche of a credit basket and other structured credit products. In Section 4 we introduce a static version of the model. We discuss numerical implementation of the static model and perform its calibration to the market for standardized CDX and iTRAXX tranches. In Section 5 we discuss possible applications of the static model to pricing bespoke tranches. In Section 6 we briefly summarize our findings.

## 2 A structural CDS model

We begin with a brief review of CDS pricing. Historically, two complementary approaches to pricing CDSs have been developed, namely the structural and reduced form approaches. In the structural framework originated by Merton (1974) and extended by Black & Cox (1976), Zhou (2001), and many others, one analyzes the evolution of the value of an individual firm (which is generally unobservable). The firm defaults if its value falls below a certain barrier, and survives otherwise. Thus, default is a predictable process. It was realized early on that this predictability makes the explanation of the short end of the CDS curve within the structural framework impossible unless jumps (cf. Zhou (2001), Lipton (2002)) or curvilinear barriers (cf. Hyer *et. al* (1998)) are incorporated. In the reduced form framework originated by Jarrow & Turnbull (1995) default is viewed as an unpredictable event whose arrival is described by a Poisson process. While the reduced form framework is capable of producing realistic short-term spreads, by construction it lacks the predictive power of the structural approach. Hence, both approaches have pros and cons. In the present paper we use the structural framework.

A typical structural model for the evolution of the log-value of the firm has the form:

$$dx = [f(t,x) - \varpi_k \lambda_k]\, dt + g(t,x)\, dW(t) + \sum_k j^{(\xi_k, p_k, q_k)} dN_k \tag{1}$$

$$x = \ln\left(\frac{V}{V_0}\right) \tag{2}$$

Here $W(t)$ is a Wiener process and $N_k(t)$ are Poisson processes with intensities $\lambda_k$. The distribution of jumps $j^{(\xi_k, p_k, q_k)}$ is assumed to be shifted exponential, with parameters satisfying the following conditions: $\xi_k = \pm 1, p_k > 1$ if $\xi_k = 1$, $p_k > 0$ if $\xi_k = -1$. The corresponding probability densities have the form

$$\phi^{(\xi,p,q)}(j) = \begin{cases} pe^{-\xi p(j-q)}, & \xi(j-q) \geq 0 \\ 0, & \xi(j-q) < 0 \end{cases} \tag{3}$$

while $\varpi_k$ are defined as follows

$$\varpi_k = \mathbb{E}\left\{e^{j^{(\xi_k, p_k, q_k)}} - 1\right\} \tag{4}$$

In principle, we can have $q \to -\infty$ which corresponds to the case of the value of the firm jumping to zero. Thus, the log-value of the firm is governed by a combination of a diffusion process and several Poisson processes with exponentially distributed jumps. The firm defaults if the value $x(t)$ crosses a (generally time-dependent) barrier $b(t)$. Along a particular trajectory the firm survives until a certain time $t$ provided that

$$y(t) = \max_{0<t'\leq t} \{b(t') - x(t')\} < 0 \tag{5}$$

Put differently, the firm survives provided that

$$H(-y(t)) = 1 \tag{6}$$

where $H(y)$ is the Heaviside function. Hence, survival probability $Q(0,t)$ can be represented in the form

$$Q(0,t) = \mathbb{E}\{H(-y(t))\} \tag{7}$$

We emphasize that by construction $-y(t)$ is monotonically decreasing, so that $Q(0,t)$ is monotonically decreasing as well. We say that equation (5) augments equation (1) - a general augmentation procedure is discussed in detail in Lipton (2001).

When all the relevant parameters are constant, the problem can be solved analytically via the Laplace transform (Lipton (2002), Hilberink and Rogers (2002)). In general, however, the analytical approach does not work, and numerical methods are required to solve the problem. A judicious combination of finite differences for partial and ordinary differential equations allows one to build a fast and accurate numerical scheme. This is a special feature of exponentially distributed jumps rather than the more familiar Gaussian jumps; details will be reported elsewhere.

# 3 A dynamic model

Although the structural approach works quite well for individual names, its extension to baskets of different names has generally not been entirely successful. In essence, the problem's dimensionality is too high, so that the problem cannot be handled in a computationally efficient way within a pure dynamic framework. Therefore, we seek to reduce the dimensionality of the model while retaining necessary correlation among individual names. Such a reduction can be achieved in a factor framework. Traditionally, this reduction is accomplished by constructing an affine model. We propose to drop affinity altogether, and modify other elements as appropriate.

More specifically, we follow Lipton (2006) and model the collective behavior of a credit basket as follows. We assume that there is a common factor describing the evolution of the state of the world which is governed by a jump-diffusion process of the form given in Eq. (1), and a cumulative factor $y$ which is governed by a non-stochastic equation of the form

$$dy = F(t,x)\,dt, \qquad F > 0 \tag{8}$$

Survival probabilities of individual names from time $t$ until time $T$, which are denoted by $Q^{(i)}(t,T)$, $1 \leq i \leq N$, where $N$ is the total number of names in the basket, are defined as follows

$$Q^{(i)}(t,T) = \mathbb{E}_t\left\{G\left(y(T) + \Theta^{(i)}(T)\right)\right\} \tag{9}$$

We assume that $G(y)$ is a logit function:

$$G(y) = \frac{1}{1+e^{y}} \tag{10}$$

which can be viewed as a diffused version of the reflected Heaviside function used in equation (6). To calibrate the model to individual names, we need to solve the following pricing PDE:

$$
\begin{aligned}
& Q_t^{(i)}+\left[f(t,x)-\sum_k \varpi_k \lambda_k\right] Q_x^{(i)}+\tfrac{1}{2} g^2(t,x)\, Q_{xx}^{(i)}+ \\
& \sum_k \lambda_k p_k \int\limits_{\min\{q_k,\xi_k\infty\}}^{\max\{q_k,\xi_k\infty\}} Q^{(i)}(x+j)\, e^{-\xi_k p_k (j-q_k)} dj+ \\
& F(t,x)\, Q_y^{(i)}-\sum_k \lambda_k Q^{(i)}=0
\end{aligned} \tag{11}
$$

$$
Q^{(i)}(T,x,y)=\frac{1}{1+e^{y+\Theta^{(i)}(T)}} \tag{12}
$$

Now our choice of the ansatz becomes more apparent. Indeed, because of our ansatz for survival probability, we can solve the same pricing equation for all names, namely:

$$
\begin{aligned}
& Q_t+\left[f(t,x)-\sum_k \varpi_k \lambda_k\right] Q_x+\tfrac{1}{2} g^2(t,x)\, Q_{xx}+ \\
& \sum_k \lambda_k p_k \int\limits_{\min\{q_k,\xi_k\infty\}}^{\max\{q_k,\xi_k\infty\}} Q(x+j)\, e^{-\xi_k p_k (j-q_k)} dj+ \\
& F(t,x)\, Q_y-\sum_k \lambda_k Q=0
\end{aligned} \tag{13}
$$

$$
Q(T,x,y)=\frac{1}{1+e^{y}} \tag{14}
$$

We calibrate $\Theta^{(i)}(T)$ to CDS spreads by solving an algebraic equation rather than a PDE:

$$
Q\left(0,0,\Theta^{(i)}(T)\right)=Q^{(i)}(0,T), \qquad 1\leq i\leq N \tag{15}
$$

Once calibration to individual names is performed, we can apply the usual recursion and calculate the probability of loss of exactly $K$ names, $0\leq K\leq N$, $P(t,y,K)$, conditional on $y$. We can then solve the pricing equation backward and find the expected losses for individual tranches at time 0. In order to price senior tranches rare but large jumps are necessary. Our numerical experiments, which will be reported elsewhere, show that the dynamic model can be adequately calibrated to the market.

When derivatives explicitly depending on the number of defaults, such as leveraged super-senior (LSS) tranches, are considered, the $x,y$ dynamics requires augmentation with the dynamics of the number of defaulted names $K$. Since we are dealing with a "pure birth" process, we can use the well-known results due to Feller (1970) and others and obtain the following expression for the one-step transition probability:

$$M(t,x,y,K) = \frac{-\sum_{K'=0}^{K}\left[P_t(t,y,K') + F(t,x)P_y(t,y,K')\right]}{P(t,y,K)} = \frac{\sum_{K'=K+1}^{K_{\max}}\left[P_t(t,y,K') + F(t,x)P_y(t,y,K')\right]}{P(t,y,K)} \quad (16)$$

The corresponding backward Kolmogoroff equation has the form:

$$V_t + \left[f(t,x) - \sum_k \varpi_k \lambda_k\right] V_x + \tfrac{1}{2} g^2(t,x) V_{xx} + \sum_k \lambda_k |p_k| \int_{\min\{0,\xi_k\infty\}}^{\max\{0,\xi_k\infty\}} V(x+j) e^{-|p_k|j} dj + F(t,x) V_y + M(t,x,y,K)\left[V(K+1) - V(K)\right] - \sum_k \lambda_k V = 0 \quad (17)$$

If need occurs, we can consider a multi-factor extension of the above model.

# 4 A static model

To make the above arguments more transparent, we introduce a simplified version of the model and apply it to CDX8 and iTRAXX7 in May 2007. On one such date, the market data for CDX8 and iTRAXX7 has the form

Table 1 near here

We can extract expected tranche losses by balancing default and payment legs in the usual manner. For simplicity we assume that the recovery rate $R$ is fixed at 40% (it is not difficult to consider non-constant recoveries). We number tranches via superscripts $^{(k)}$, $k = 1, ..., 6$ with $k = 1$ corresponding to equity tranche, $k = 2$ corresponding to mezzanine tranche, etc., $k = 6$ corresponding to the index itself, and introduce the coefficient $\phi^{(k)}$, such that

$$\phi^{(k)} = 1, \qquad k = 1, ..., 5, \qquad \phi^{(6)} = \frac{1}{1-R} \quad (18)$$

Let $t_i$ be the times of coupon payments, and $n_m$, $m = 1, 2, 3$, be chosen in such a way that $t_{n_1} \equiv T_1 = 5y$, $t_{n_2} \equiv T_2 = 7y$, $t_{n_3} \equiv T_3 = 10y$. In terms of the tranche notional amounts $N^{(k)}$ and their expected losses $L_i^{(k)}$, the default leg $DL_m^{(k)}$ and premium leg $PL_m^{(k)}$ can be written respectively as

$$DL_m^{(k)} = \sum_{i=1}^{n_m} DF\left(\frac{t_i + t_{i-1}}{2}\right)\left(L_i^{(k)} - L_{i-1}^{(k)}\right) \quad (19)$$

$$PL_m^{(k)} = uf_m^{(k)} N^{(k)} + s_m^{(k)} \sum_{i=1}^{n_m} (t_i - t_{i-1}) DF(t_i) \left( N^{(k)} - \phi^{(k)} L_i^{(k)} \right) \tag{20}$$

Here $uf_m^{(k)}, s_m^{(k)}$ are the upfront payment and spread for a given tranche, respectively. We interpolate expected losses linearly on the intervals $[T_1, T_2]$, and $[T_2, T_3]$,

$$L_i^{(k)} = \frac{\left(t_i - T_{n_{m-1}}\right) \bar{L}_m^{(k)} + (T_{n_m} - t_i) \bar{L}_{m-1}^{(k)}}{\left(T_{n_m} - T_{n_{m-1}}\right)}, \qquad n_{m-1} + 1 \leq i \leq n_m \tag{21}$$

and as a power on the interval $[0, T_1]$

$$L_i^{(k)} = \left( \frac{t_i}{T_1} \right)^p \bar{L}_1^{(k)}, \qquad 1 \leq i \leq n_1 \tag{22}$$

where the value of $p$ is typically close to 2. By equating the legs in (19), (20), we obtain a system of lower-triangular equations which we can solve for the expected tranche losses $\bar{L}_m^{(k)}$. These losses (per dollar) for CDX8 and iTRAXX7 are shown in Table 2:

Table 2 near here

In the spirit of our previous consideration, we introduce a market factor with four distinct values $y_l(T)$,

$$y_0(T) = 0, y_1(T), y_2(T), y_3(T) = \infty \tag{23}$$

occurring with probabilities $\pi_l(T)$,

$$\pi_0(T) + \pi_1(T) + \pi_2(T) + \pi_3(T) = 1 \tag{24}$$

We assume again that the conditional survival probabilities of individual names in different states of the world are given by the logit expression which we used in the dynamic setting,

$$Q_l^{(i)}(T) = \frac{1}{1 + e^{y_l(T) + \Theta^{(i)}(T)}}, \qquad l = 0, ..., 3 \tag{25}$$

and calibrate model parameters to match the market. It is evident from equations (23) and (24) that we have five free parameters for each maturity $T$. It is also clear that the last state of the world ($l = 3$) describes the situation of a catastrophic meltdown when all the names in the basket default simultaneously.

To calibrate the model to survival probabilities of individual names, we solve the following equation for $\Theta^{(i)}(T)$:

$$\frac{\pi_0(T)}{1 + e^{y_0(T) + \Theta^{(i)}(T)}} + \frac{\pi_1(T)}{1 + e^{y_1(T) + \Theta^{(i)}(T)}} + \frac{\pi_2(T)}{1 + e^{y_2(T) + \Theta^{(i)}(T)}} = Q^{(i)}(T) \tag{26}$$

Substitution $\exp\left\{\Theta^{(i)}(T)\right\} = Z^{(i)}(T)$ turns the above equation into a cubic equation which can be solved via the Cardano formula. Provided that

$$Q^{(i)}(T) < 1 - \pi_3(T) = \pi_0(T) + \pi_1(T) + \pi_2(T) \tag{27}$$

so that survival probability of a name is compatible with the doomsday scenario, there is always a single positive root $Z_+^{(i)}(T)$, which yields $\Theta^{(i)}(T) = \ln\left\{Z_+^{(i)}(T)\right\}$. Next, we use the standard recursion in order to calculate default probability distributions in each of the four states of the world (in the catastrophic states the corresponding distribution is a delta function), and finally expected losses for all tranches.

By adjusting $\pi_l$ and $y_l$ we can achieve very good and often exact calibrations to the market for both CDX8 and iTRAXX7, and reproduce the losses shown in Table 2 exactly. The corresponding parameters for CDX8 and iTRAXX7 have the form:

Table 3 near here

They are graphically represented in Figure 1.

Figure 1 near here

We do not show the traditional table of model-market differences here since a set of machine zeroes is rather monotonous. In Figure 2 we display the overall loss distributions for CDX8 and iTRAXX7; in both cases for $T = 7y$.

Figure 2 near here

We emphasize that in our formulation the states of the world are different for different maturities (although their total number stays the same). Thus, we neither achieve nor require monotonicity for survival probabilities in a particular state (except for the catastrophic state).

Once the parameters of the model are determined we can calculate the entire loss distribution surface which is shown in Figures 3, 4 for CDX8. For iTRAXX7 the corresponding surface has a similar form.

Figures 3, 4 near here

It is obvious that our construction of the loss distribution for a given time does not require any interpolation and hence is arbitrage-free. Figure 5 shows that there is also no arbitrage across time.

Figure 5 near here

It is worth noting that in the limit of a large homogeneous portfolio, the distribution of relative losses is atomic in nature and localized at four distinct points $0 < \alpha_0 < \alpha_1 < \alpha_2 < \alpha_3 = 1$, on the interval $[0, 1]$. This distribution is to be contrasted with the famous continuous distribution introduced by Vasicek for the Gaussian copula.

# 5 Pricing of bespoke tranches

One of the simplest applications of the static model is the calculation of the BECs for tranchelets. Since we know the entire loss surface, we can calculate the expected losses for any tranchelet and hence find the corresponding BEC. We show BECs for 1% tranchelets for $T = 7y$ in Figure 6.

Figure 6 near here

Pricing of bespoke tranches requires additional assumptions. When iTRAXX7 parameters are used for CDX8 and vice versa the corresponding calibration errors have the form:

Table 4 near here

The fact that these errors are relatively modest suggests that (within reason) we can use the same parameters to price standard and bespoke tranches. For instance, we might use CDX parameters throughout; a more prudent course of action would be to mix parameters to reflect the actual mix of the basket.

More complicated instruments (such as LSS tranches and the like) can be priced in the framework of the complete model.

# 6 Conclusions

In this paper we demonstrated how to construct a one-factor dynamic factor model for credit baskets. Via a special ansatz we showed how to solve the calibration problem for individual names without the need for analytically tractable (but not necessarily financially intuitive) dynamics. We then proposed a simplified static version of the model, and showed that even this simplified version is able to reproduce the market exactly. We used the calibrated simplified model to analyze the structure of loss distributions implied by market quotes. As a result we obtained a financially meaningful “completion” of the break-even coupon surface.

In its full form, the model can be used to price a variety of exotic credit products (such as forward starting tranches, LSS tranches, counterparty haircuts, etc.), and to build dynamic hedges for such structures.

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| Tranche\Tenor | 3y | 5y | 7y | 10y |
|---|---|---|---|---|
| CDX Index | 19.50 | 35.00 | 47.00 | 61.00 |
| $0-3\%$ | – | 24.06% | 40.50% | 51.81% |
| $3-7\%$ | – | 95.50 | 221.63 | 492.50 |
| $7-10\%$ | – | 21.00 | 49.00 | 122.63 |
| $10-15\%$ | – | 9.50 | 23.00 | 56.50 |
| $15-30\%$ | – | 3.50 | 8.75 | 17.25 |
| $30-100\%$ | – | 1.38 | 3.50 | 5.88 |
| iTRAXX Index | 10.30 | 20.38 | 29.38 | 40.48 |
| $0-3\%$ | – | 6.76% | 21.08% | 35.50% |
| $3-6\%$ | – | 41.25 | 106.72 | 298.75 |
| $6-9\%$ | – | 10.25 | 27.64 | 87.18 |
| $9-12\%$ | – | 4.18 | 13.26 | 40.10 |
| $12-22\%$ | – | 1.93 | 5.22 | 12.51 |
| $22-100\%$ | – | – | – | – |

Table 1. Market data for CDX8 and iTRAXX7 on 30/05/2007.

| Tranche\Tenor | 3y | 5y | 7y | 10y |
|---|---|---|---|---|
| CDX Index | 0.0061 | 0.0186 | 0.0350 | 0.0647 |
| $0-3\%$ | 0.1679 | 0.5133 | 0.7739 | 0.9708 |
| $3-7\%$ | 0.0165 | 0.0504 | 0.1665 | 0.5077 |
| $7-10\%$ | 0.0037 | 0.0112 | 0.0380 | 0.1402 |
| $10-15\%$ | 0.0017 | 0.0051 | 0.0180 | 0.0656 |
| $15-30\%$ | 0.0006 | 0.0019 | 0.0069 | 0.0200 |
| $30-100\%$ | 0.0002 | 0.0007 | 0.0028 | 0.0067 |
| iTRAXX Index | 0.0033 | 0.0109 | 0.0222 | 0.0440 |
| $0-3\%$ | 0.0993 | 0.3241 | 0.5752 | 0.8453 |
| $3-6\%$ | 0.0068 | 0.0221 | 0.0824 | 0.3303 |
| $6-9\%$ | 0.0017 | 0.0055 | 0.0217 | 0.1019 |
| $9-12\%$ | 0.0007 | 0.0022 | 0.0105 | 0.0474 |
| $12-22\%$ | 0.0003 | 0.0010 | 0.0041 | 0.0147 |
| $22-100\%$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 2. Losses per dollar for various tranches of CDX8 and iTRAXX7.

| Param\Tenor | 3y | 5y | 7y | 10y |
|---|---|---|---|---|
| $\pi_0$(CDX) | 0.9776 | 0.9391 | 0.8033 | 0.6397 |
| $\pi_1$ | 0.0207 | 0.0558 | 0.1663 | 0.2893 |
| $\pi_2$ | 0.0013 | 0.0040 | 0.0237 | 0.0549 |
| $\pi_3$ | 0.0003 | 0.0010 | 0.0067 | 0.0161 |
| $y_1$ | 2.8506 | 1.6351 | 1.0149 | 0.6896 |
| $y_2$ | 4.3553 | 3.1101 | 2.0591 | 1.7759 |
| $\pi_0$(iTRAXX) | 0.9904 | 0.9696 | 0.8995 | 0.6547 |
| $\pi_1$ | 0.0090 | 0.0282 | 0.0892 | 0.2955 |
| $\pi_2$ | 0.0005 | 0.0017 | 0.0085 | 0.0427 |
| $\pi_3$ | 0.0001 | 0.0005 | 0.0028 | 0.0072 |
| $y_1$ | 3.1179 | 1.9090 | 1.3220 | 0.9328 |
| $y_2$ | 4.4254 | 3.2209 | 2.4541 | 2.0030 |

Table 3. Calibration parameters for CDX8 and iTRAXX7.

| Tranche\Tenor | 3y | 5y | 7y | 10y |
|---|---|---|---|---|
| CDX Index | 0.1 | 0.0 | 0.0 | 0.0 |
| $0-3\%$ | – | 2.377% | 4.429% | 2.219% |
| $3-7\%$ | – | −20.5 | −0.8 | 33.3 |
| $7-10\%$ | – | 6.7 | 12.5 | 54.4 |
| $10-15\%$ | – | −2.7 | −4.3 | −3.4 |
| $15-30\%$ | – | −1.2 | −3.1 | −4.1 |
| $30-100\%$ | – | −1.0 | −2.2 | −3.2 |
| iTRAXX Index | −0.1 | 0.0 | 0.0 | 0.0 |
| $0-3\%$ | – | −1.512% | −3.615% | −3.612% |
| $3-6\%$ | – | 2.5 | −27.6 | −68.4 |
| $6-9\%$ | – | −0.2 | 2.4 | −20.7 |
| $9-12\%$ | – | 1.9 | −1.5 | 0.5 |
| $12-22\%$ | – | 1.1 | 3.9 | 3.1 |
| $22-100\%$ | – | 1.7 | 0.0 | 0.0 |

Table 4. Model-market discrepancies when iTRAXX7 parameters are used to price CDX8 and vice versa.

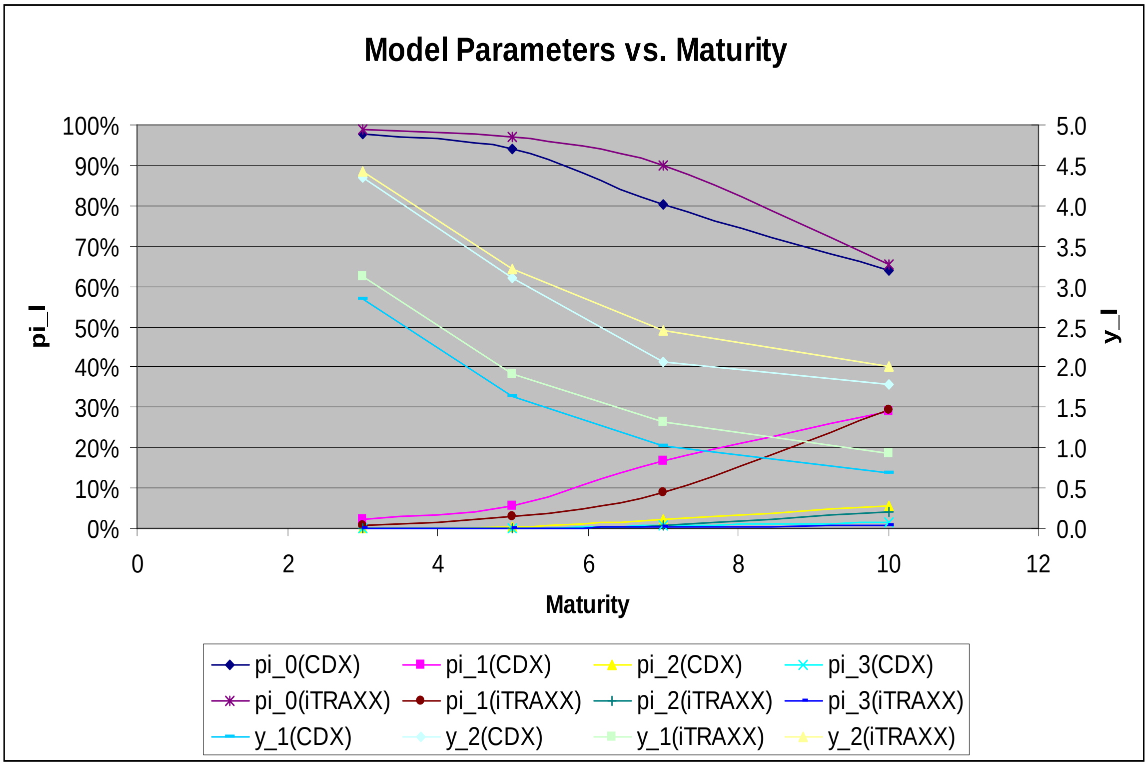


Figure 1: Model parameters for CDX8 and iTRAXX7 as functions of maturity.

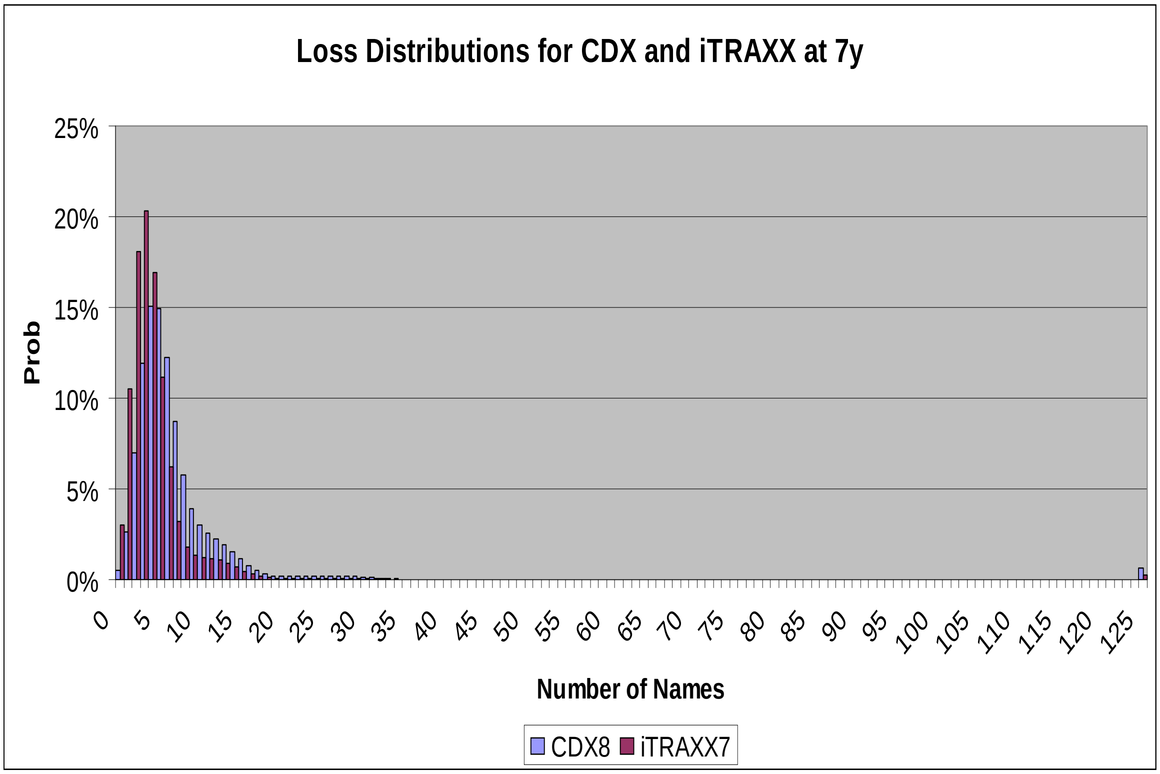


Figure 2: Loss distributions for CDX8 and iTRAXX7 at $T = 7y$.

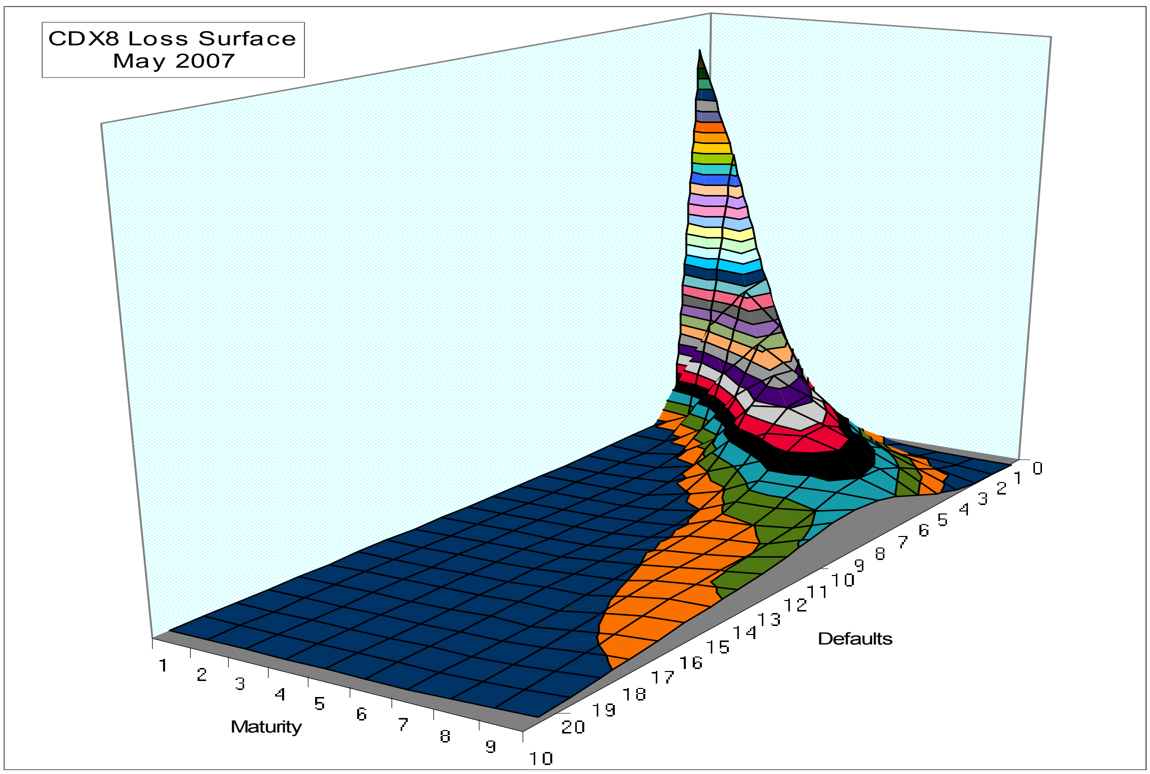


Figure 3: Loss distribution surface for CDX8.

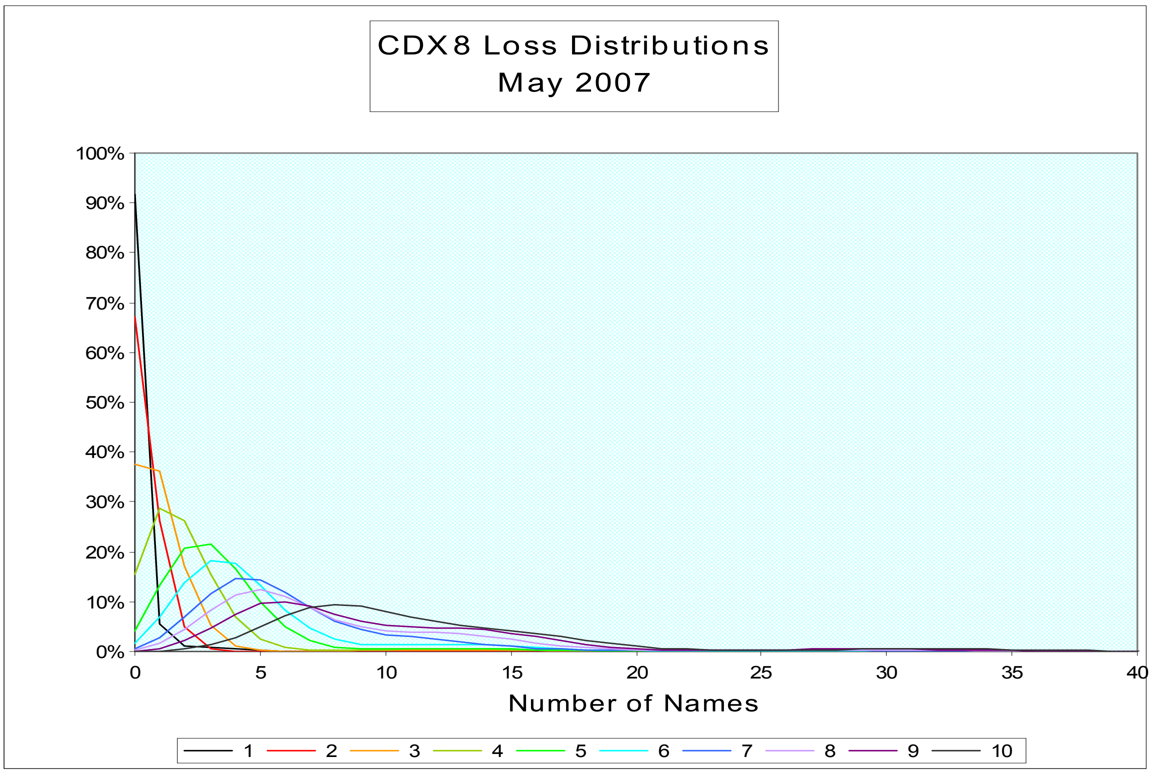


Figure 4: Cross-sections of loss distribution surface for CDX8.

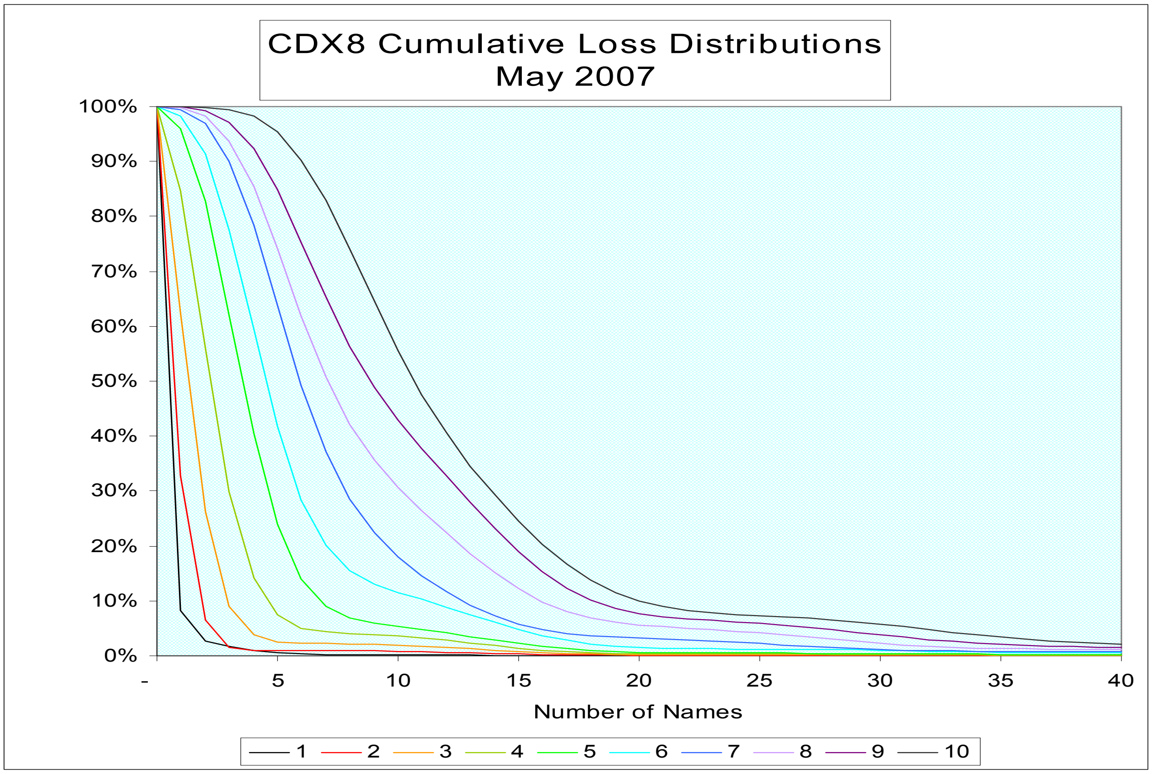


Figure 5: Cross-sections of cumulative loss distribution surface for CDX8.

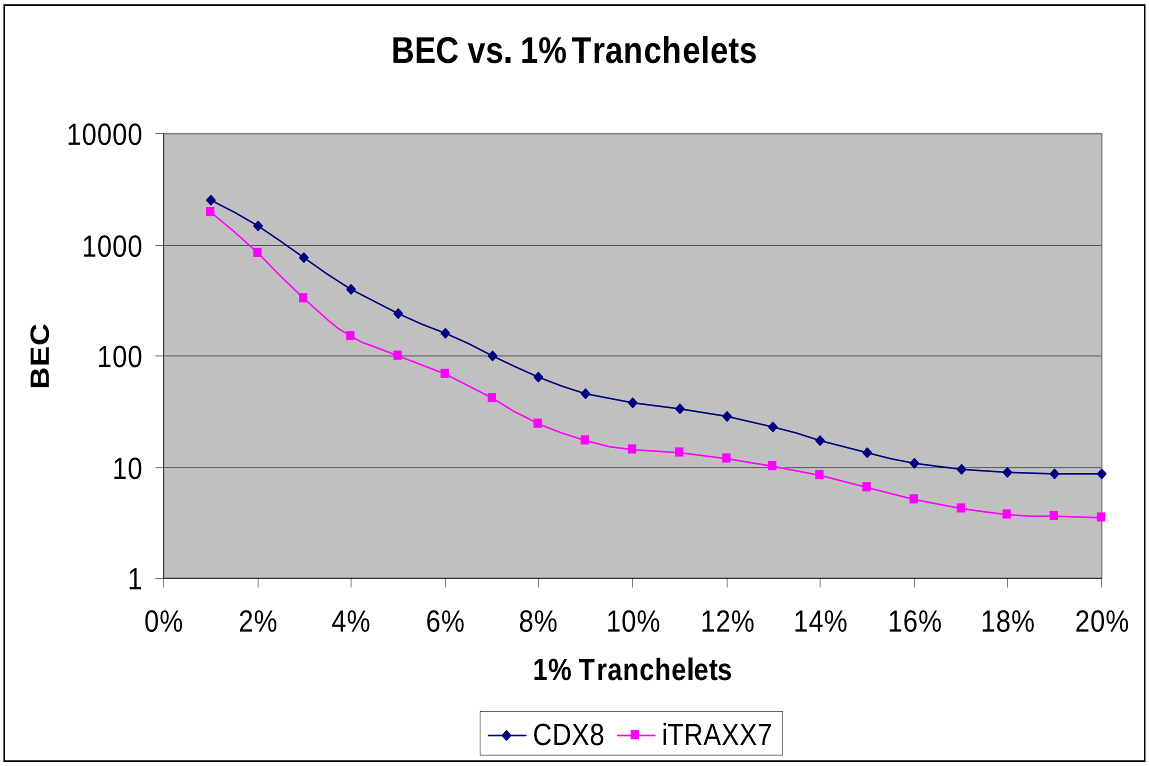


Figure 6: BEC for 1% tranchelets for CDX8 and iTRAXX7, $T = 7y$.